



## 26 BACKGROUND

### 27 Natural Frequency Predictions

28 TD5 Chapter 2 and DG11 Chapter 3 provide a manual calculation method for the fundamental  
29 natural frequency, typically referred to as “the natural frequency.” Most floors have joists or beams  
30 supported by girders, so the fundamental mode shape has two-way bending. The manual method begins  
31 with computing the natural frequency of an idealized joist or beam mode,  $f_j$ , and the natural frequency of  
32 an idealized girder mode,  $f_g$ , using the following equation:

$$33 \quad f_{j,g} = 0.18 \sqrt{\frac{g}{\Delta_{j,g}}} \quad (\text{TD5 Eq. 2.3 / DG11 Eq. 3-3})$$

34 where  $g$  is the acceleration due to gravity on the Earth’s surface, 386 in./s<sup>2</sup> and  $\Delta_{j,g}$  is the deflection of  
35 joist or girder, in., due to slab weight, framing weight, and superimposed dead and live loads  
36 recommended in DG11 Chapter 3. These are unfactored best estimates of the loads that will be present  
37 during a vibration event, not higher-end values used for strength or stiffness design.

38 In TD5 Chapter 2 and DG11 Chapter 3, the floor bay frequency is predicted by combining the joist  
39 or beam and girder natural frequencies using the Dunkerley equation, shown below. This combined mode  
40 frequency was used in the original development of the acceleration prediction equations, so it must be  
41 used when predicting accelerations in TD5 Chapter 3 and DG11 Section 4.1. However, it underpredicts  
42 the natural frequency by about 25% (Pabian et al., 2013).

$$43 \quad f_n = 0.18 \sqrt{\frac{g}{\Delta_j + \Delta_g}} \quad (\text{TD5 Eq. 2.4 / DG11 Eq. 3-4})$$

44 A more accurate prediction of the floor bay natural frequency results from taking the minimum of  
45 the joist or beam and girder frequencies as shown below (Pabian et al., 2013). Herein, this frequency is  
46 referred to as  $f_{n1}$ , a variable name that does not appear in TD5 or DG11. This prediction cannot be used in  
47 the TD5 Chapter 3 and DG11 Section 4.1 acceleration predictions, but it can be used for other purposes.

$$48 \quad f_{n1} = \min(f_j, f_g) \quad (\text{DG11 Eq. 6-1})$$

49 The finite element analysis methods in DG11 Chapter 7 also result in accurate predictions of natural  
50 frequencies.

## 51 **Walking**

52 Walking on floors can cause resonant build-ups. To understand why, we need to look at the dynamic  
53 forces due to walking.

54 The dynamic force due to repeated footsteps or other human-induced forces is represented by a  
55 Fourier series of the following form. Each term in the summation is a sinusoid called a harmonic.

$$56 \quad F(t) = Q + \sum_{i=1}^N \alpha_i Q \sin(2\pi i f_{step} t - \phi_i) \quad (\text{DG11 Eq. 1-1})$$

57 where

58  $i$  = harmonic number

59  $N$  = number of harmonics with significant force amplitude ( $N = 4$  for walking)

60  $\alpha_i$  =  $i^{\text{th}}$  harmonic dynamic coefficient, i.e., the ratio of harmonic force amplitude to bodyweight

61  $f_{step}$  = step frequency, Hz

62  $\phi_i$  =  $i^{\text{th}}$  harmonic phase lag, radians

63 The bodyweight,  $Q$ , is 157 lb in TD5 Chapter 3 and DG11 Section 4.1, which stem from Allen and  
64 Murray (1993). In most new formulations in DG11,  $Q$  is 168 lb.

65 For the  $i^{\text{th}}$  harmonic, the frequency is  $if_{step}$  and the sinusoidal force amplitude is  $\alpha_i Q$ . The following  
66 table summarizes the harmonics of the force caused by a 157 lb walker.

67 Table 1. Walking Force Harmonics

Harmonic, $i$	$if_{step}$ range (Hz)	$\alpha_i$	$\alpha_i Q$ (lb)
1	1.6 – 2.2	0.5	78.5
2	3.2 – 4.4	0.2	31.4
3	4.8 – 6.6	0.1	15.7
4	6.4 – 8.8	0.05	7.85

68

69

70 Key takeaways from Table 1:

- 71 • If the natural frequency is below about 2.2 Hz, the first harmonic might cause resonance.  
72 The first harmonic has a high force amplitude, so this would often result in high  
73 accelerations.
- 74 • If the natural frequency is between 2.2 Hz and about 9 Hz, one of the higher harmonics  
75 could cause resonance. Such floors are referred to in DG11 as low frequency floors (LFFs).  
76 Most steel-framed floors are LFFs. The higher harmonics have lower force amplitudes, so it  
77 is typically straightforward to design these to satisfy human comfort vibration limits.

78 Another key piece of information: The walking acceleration predictions in TD5 Equation 3.1 and  
79 DG11 Equation 4-1 are based on a curve fit of the second through fourth harmonic dynamic coefficients,  
80 as explained above DG11 Equation 2-4. The underlying assumption is that the first harmonic will not  
81 cause resonance; this is equivalent to assuming the natural frequency exceeds 2.2 Hz.

82 It is important to prevent the first harmonic from causing resonance. Walking at 2.2 Hz will include  
83 some energy at slightly higher frequencies, say 2.3-2.4 Hz. Also, walking at slightly above 2.2 Hz, and  
84 short instances of running or bouncing in place occasionally occur, even on non-gym floors. There should  
85 be a cushion between 2.2 Hz and the natural frequency, so 3 Hz seems like a prudent lower limit on the  
86 natural frequency.

87

88 **Vandal Jumping**

89 When occupants perceive very low frequency floor vibration, they might form a group and  
90 mischievously jump or bounce in place to intentionally cause high vibration levels. This activity is called  
91 vandal jumping or rogue jumping. The group attempts to synchronize the jumping frequency with the  
92 frequency of vibration, which is predominately at the natural frequency.

93 Vandal jumping is similar to aerobics. The second and third columns in Table 2 list the harmonic  
94 frequency ranges and dynamic coefficients that are recommended in TD5 Chapter 4 and DG11 Chapters 5  
95 and 7. The size of the group is unknown, but three vandal jumpers is a reasonable assumption. The last  
96 column lists the harmonic force amplitudes due to a group of three 168 lb vandal jumpers.

97 Table 2. Vandal Jumping Force Harmonics

Harmonic, $i$	$if_{step}$ range (Hz)	$\alpha_i$	$nQ\alpha_i$ (lb)
1	2.0 – 2.75	1.5	760
2	2.75 – 5.5	0.6	302
3	5.5 – 8.25	0.1	50

98 Key takeaways from Table 2:

- 99 • The jumping frequency, equal to the first harmonic frequency, has a maximum value of 2.75  
100 Hz.
- 101 • The harmonic force magnitudes due to three vandal jumpers are much higher than the forces  
102 due to walking, shown in Table 1.

103 A group of vandal jumpers is like a small aerobics class on a floor that was not designed for  
104 aerobics. For aerobics floors, the strategy is to increase the natural frequency beyond 6 Hz so that neither  
105 the first nor second harmonic can cause resonance. An aerobics floor with a natural frequency below 3 Hz  
106 has very little chance of acceptable vibration performance. In a nutshell, this is why it is critical to prevent  
107 vandal jumping.

108 Without vibration testing equipment, a group can only attempt to synchronize the jumping frequency  
109 with the natural frequency. They cannot intentionally synchronize the higher harmonic frequencies,  $2f_{step}$

110 or  $3f_{step}$ , with the natural frequency. For example, if the natural frequency is 5 Hz, they would need to  
111 jump at  $5 \text{ Hz} / 2 = 2.5 \text{ Hz}$ , which would require jumping every other time the floor oscillates.

112 Thus, if the natural frequency exceeds 3 Hz, the group will not attempt to – or be able to –  
113 synchronize the jumping frequency and natural frequency. This is the most important reason for designing  
114 floors so that the natural frequency is at least 3 Hz.

## 115 **NATURAL FREQUENCY LOWER LIMIT RECOMMENDATION**

116 Due to the reasons explained above, the floor natural frequency should be at least 3 Hz for the vast  
117 majority of floors. The first step is to determine if the floor satisfies this design check. As explained  
118 above,  $f_{n1}$  is an accurate prediction of the natural frequency. Including consideration of a potential 10%  
119 overprediction of  $f_{n1}$ , the frequency check is:

$$120 \quad \begin{aligned} 0.9f_{n1} &\geq 3 \text{ Hz} \\ f_{n1} &\geq 3.3 \text{ Hz} \end{aligned} \quad (1)$$

121 Equation 1 approximately corresponds to limiting the traditional Dunkerley  $f_n$  to 2.5 Hz, so it is more  
122 lenient than the traditional check.

123 Note that the Dunkerley  $f_n$  must be used in the acceleration predictions in TD5 Chapter 3 and DG11  
124 Section 4.1.

125

126 **EVALUATION OF ULTRA-LOW FREQUENCY FLOORS**

127 Some floors not conforming to Equation 1, referred to herein as ultra-low frequency floors (ULFFs)  
128 might be satisfactory for vibration relative to human comfort. This is especially true of those with  
129 extremely large masses, high damping ratios, and lenient tolerance limits.

130 The following ULFF evaluation criterion applies when  $f_{n1} \geq 2.5$  Hz. When the natural frequency is  
131 below that, more elaborate analyses would be required.

132 An ULFF should have satisfactory vibration performance when Equation 2 is satisfied for walking  
133 and for vandal jumping, with the latter being more severe by an order of magnitude.

134 
$$\frac{a_p}{g} \leq \frac{a_o}{g} \quad (2)$$

135 Equation 2 is extremely difficult to satisfy for vandal jumping, even with massive floors, high  
136 damping ratios, and lenient tolerance limits. An engineer might decide to evaluate the floor for walking  
137 and ignore vandal jumping based on engineering judgment for a particular project. However, ignoring  
138 vandal jumping would be a departure from design recommendations dating back to the first edition of  
139 Design Guide 11 in 1997, so it is not recommended in general.

140 The tolerance limit for human comfort,  $a_o$ , is from TD5 Figure 1.1 and DG11 Figure 2-1. For each  
141 occupancy, the limit is most stringent between 4 Hz and 8 Hz and is more lenient below 4 Hz. However,  
142 it is recommended that the most stringent range be used below 4 Hz also. Because this article is limited to  
143 floors subjected to walking, the set of limits is reduced to the following:

- 144 • For quiet areas with seated occupants, such as offices, residences, churches, and schools, the  
145 limit is 0.5%g.
- 146 • For open areas with higher ambient noise and with affected occupants who are  
147 predominately standing, the limit is 1.5%g.
- 148 • Limits for other scenarios would need to be established using engineering judgment.

149 Manual calculation methods for computing the predicted acceleration,  $a_p$ , for walking or vandal  
150 jumping on ULFF are given in the following sections. Walking will not control; its equation is provided  
151 for cases in which the engineer chooses to ignore vandal jumping.

152 Similar finite element analysis methods could be derived also.

### 153 **Acceleration due to Walking on Ultra-Low Frequency Floors**

154 The predicted acceleration of an ULFF caused by walking is derived using the single degree of  
155 freedom (SDOF) approach used in DG11 Equation 2-5. For an ULFF, the key assumption is that the first  
156 harmonic of the walking force causes resonance. The resulting equation is:

$$\begin{aligned} a_p &= R \frac{\text{Sinusoidal Force Amplitude}}{2\beta M} \\ &= \frac{RQ\alpha_1}{2\beta M} \\ &= \frac{(0.5)(157 \text{ lb})(0.5)}{2\beta(0.5W / g)} \\ &= \frac{39 \text{ lb}}{\beta W} g \end{aligned} \tag{3}$$

158 where

159  $R$  = reduction factor, 0.5 for floors

160  $\alpha_1$  = dynamic coefficient of the first harmonic of the walking force, 0.5 (Table 1)

161  $\beta$  = damping ratio from TD5 Table 3.2 / DG11 Table 4-2

162  $M$  = fundamental modal mass, lb-s<sup>2</sup>/in.

163  $W$  = effective weight from TD5 Equation 3.6 / DG11 Equation 4-5, lb

164 The criterion defined by Equations 2 and 3 is fairly difficult to satisfy. For example, if a floor bay  
165 supports an electronic office and a typical ceiling, so that the damping ratio is 0.025, the required  
166 effective weight,  $W$ , is 312,000 lb. Another example: if a bay supports an office and has significant full-  
167 height partitions, so that the damping ratio is 0.05, the required effective weight is 156,000 lb. Bays with  
168 such large effective weights occur in practice but are not common.

169 **Acceleration due to Vandal Jumping on Ultra-Low Frequency Floors**

170 The predicted acceleration due to vandal jumping on an ULFF is also derived using the SDOF  
171 approach. The underlying assumption is that the first harmonic will dominates the response, so there is no  
172 need to compute the responses to higher harmonics.

173 The resulting equation for the predicted acceleration caused by vandal jumping is:

$$\begin{aligned} a_p &= \frac{nQ\alpha_1}{2\beta M} \\ &= \frac{(3)(168 \text{ lb})(1.5)}{2\beta(0.5W / g)} \\ &= \frac{756}{\beta W} g \end{aligned} \tag{4}$$

175 where

176  $n$  = number of vandal jumpers

177  $Q$  = weight of one vandal jumper, lb

178  $\alpha_1$  = dynamic coefficient of the first harmonic of the force due to aerobics, 1.5 (Table 2)

179  $\beta$  = damping ratio from TD5 Table 3.2 / DG11 Table 4-2

180  $M$  = fundamental modal mass, lb-s<sup>2</sup>/in.

181  $W$  = effective weight from TD5 Equation 3.6 / DG11 Equation 4-5, lb

182 To the writer’s knowledge, there is no research that could be used to determine the number of vandal  
183 jumpers in the group. From engineering judgment, it is reasonable to consider three vandal jumpers.

184 The reference bodyweight for this equation is 168 lb, which is commonly used in DG11 for newer  
185 derivations.

186 A comparison of Equations 3 and 4 indicates vandal jumping is much more severe than walking.  
187 Thus, if vandal jumping is evaluated, there is no need to evaluate walking also.

188 The criterion defined by Equations 1 and 4 is extremely difficult to satisfy, even with extremely  
189 massive floors with high damping ratios and lenient tolerance limits.

190 **SUMMARY**

191 Practical vibration evaluation criteria for steel-framed structures are provided in the SJI Technical  
192 Digest 5 and AISC Design Guide 11. In each criterion, the primary design check is a comparison of the  
193 predicted acceleration and tolerance limit. Most criteria also state that the natural frequency should  
194 exceed a lower limit. For floors, that limit has been 3 Hz since the first edition of Design Guide 11 in  
195 1997. This article provides background on this limit and design recommendations.

196 A natural frequency below the lower limit results in the following consequences:

- 197 • The first harmonic of the walking force can cause resonance, so the typical acceleration  
198 prediction equations do not apply. Also, the predicted acceleration due to walking is  
199 substantially increased.
- 200 • A group of occupants might “vandal jump” to intentionally cause much higher accelerations.  
201 This is an extremely severe analysis case.

202 The primary recommendation is to design the floor so that the natural frequency, taken as the  
203 minimum of the joist or beam and girder natural frequencies, called  $f_{n1}$  herein, is at least 3.3 Hz. This  
204 recommendation is more lenient than the traditional check in which the Dunkerley natural frequency is  
205 compared to 3 Hz.

206 It is possible that a very massive floor with high damping and a lenient tolerance limit might be  
207 satisfactory even if  $f_{n1}$  is lower than 3.3 Hz. An evaluation criterion was given for such cases. Prediction  
208 equations suitable for manual calculations were derived for acceleration due to first harmonic walking and  
209 vandal jumping. Vandal jumping, which controls over first harmonic walking by an order of magnitude,  
210 would be very difficult to satisfy.

211 **REFERENCES**

- 212 Allen, D.E. and Murray, T.M. (1993), “Design Criterion for Vibrations Due to Walking,” *Engineering*  
213 *Journal*, 30(4), 117-129.
- 214 Murray, T.M. and Davis, B. (2015), *Vibration Analysis of Steel Joist-Concrete Floor Systems*, 2<sup>nd</sup> ed.,  
215 Technical Digest 5, Steel Joist Institute, Florence, South Carolina.
- 216 Murray, T.M., Allen, D.E., and Ungar, E.E. (1997), *Steel Design Guide Series 11: Floor Vibrations Due*  
217 *to Human Activity*, American Institute of Steel Construction, Chicago, Illinois.
- 218 Murray, T.M., Allen, D.E., Ungar, E.E., and Davis, D.B. (2016), *Vibrations of Steel-Framed Structural*  
219 *Systems Due to Human Activity*, Design Guide 11, American Institute of Steel Construction, Chicago,  
220 Illinois.
- 221 Pabian, S., Thomas, A., Davis, B., and Murray, T.M. (2013), “Investigation of Floor Vibration Evaluation  
222 Criteria Using an Extensive Database of Floors,” *Proceedings of the ASCE Structures Congress*,  
223 ASCE, Reston, VA, 2478-2486.